

Chaining methods and their application to genomic data

DaSciM seminar

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Introduction: Multi-output prediction

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Raccoon? YES



Raccoon? NO

Machine learning



Raccoon? YES



Raccoon? NO

We want to find the **best** model f :

$$X \xrightarrow{f} y$$

$f : f(X) = \hat{y}$,
such that the loss function $L(\hat{y}, y)$ is minimal.

Examples of loss functions:

- Regression: MSE, MAE
- Classification: 0/1 loss

Multi-output machine learning



Raccoon? YES

Multi-output machine learning



Raccoon? YES

Wolf? NO

Beaver? NO

Has stripes? YES

Has fur? YES

Multi-output machine learning



Raccoon? **YES** $f_1(\mathbf{X}) = y_1$

Wolf? **NO** $f_2(\mathbf{X}) = y_2$

Beaver? **NO** $f_3(\mathbf{X}) = y_3$

Has stripes? **YES** $f_4(\mathbf{X}) = y_4$

Has fur? **YES** $f_5(\mathbf{X}) = y_5$

Multi-output machine learning



Raccoon? **YES**

Wolf? **NO**

Beaver? **NO**

Has stripes? **YES**

Has fur? **YES**

$$f(X) = y$$

$$y = (y_1, y_2, y_3, y_4, y_5)$$

Multi-output machine learning



Raccoon? **YES**

Wolf? **NO**

Beaver? **NO**

Has stripes? **YES**

Has fur? **YES**

$$f(X) = y$$

$$y = (y_1, y_2, y_3, y_4, y_5)$$

Idea: to model these labels together in order to get better prediction performance

Chaining methods

Definition of a multi-output problem

Given:

Dataset $\mathcal{D} = \{(\mathbf{x}^i, \mathbf{y}^i)\}_{i=1}^N$ of N samples:

- features $\mathbf{x}^i = [x_1^i, \dots, x_M^i]$
- outputs $\mathbf{y}^i = [y_1^i, \dots, y_L^i]$

Goal:

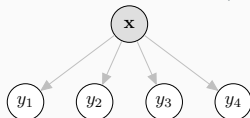
Model which outputs predictions $\hat{\mathbf{y}}^i = [\hat{y}_1^i, \dots, \hat{y}_L^i]$ having \mathcal{D} observed.

	Raccoon?	Wolf?	Beaver?	Has stripes?	Has fur?
x_1	1	0	0	1	1
x_2	1	0	0	0	1
x_3	0	0	1	0	1
x_4	0	1	0	0	1
x_5	0	0	0	1	0
x_6	?	?	?	?	?

Some approaches to multi-output problems

- Independent models (= *binary relevance* for classification):

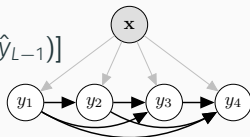
$$\hat{\mathbf{y}} = [\hat{y}_1, \dots, \hat{y}_L] = [h_1(\mathbf{x}), \dots, h_L(\mathbf{x})]$$



- Fully-cascaded chain:

$$\hat{\mathbf{y}} = [\hat{y}_1, \dots, \hat{y}_L] = [h_1(\mathbf{x}), h_2(\mathbf{x}, \hat{y}_1), \dots, h_L(\mathbf{x}, \hat{y}_1, \dots, \hat{y}_{L-1})]$$

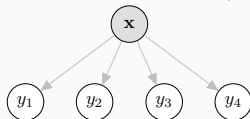
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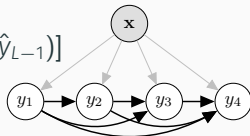
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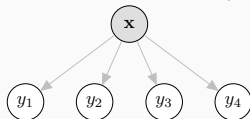


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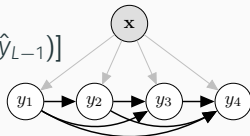
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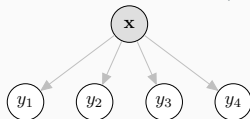


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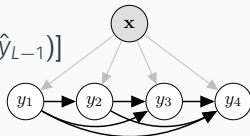
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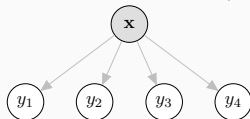


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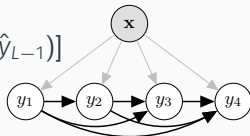
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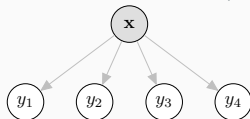


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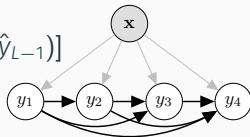
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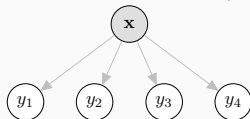


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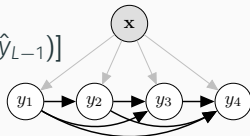
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Does the chaining approach work?

Classification

Classifier Chains have proved to be flexible and effective and have achieved state-of-the-art empirical performance

- *Classifier Chains: A Review and Perspectives*, Read et al., 2021

Regression

Regressor Chains show relatively few advantages compared to individual regression models.

State-of-the-art methods:

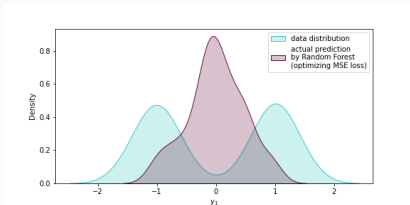
- Multi-output Decision Trees (DT)
- Multi-output Random Forests (RF)
- Independent Regressors (IR)

Regressor chains: why don't they work?

1. Inadequate choice of the loss function to optimize

Most models optimize $MSE = \frac{1}{N} \sum_{j=1}^N (y_j - \hat{y}_j)^2$.

- Example: multi-modal distribution \implies standard models may be inappropriate.



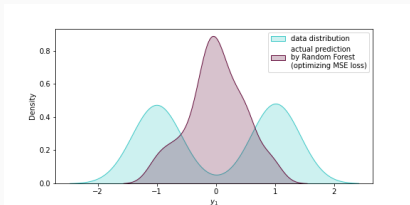
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2. Insufficient depth of the model

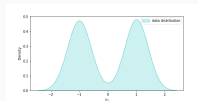
- Only one round of prediction
- Fixed cascaded order

Our improvements for Regressor Chains

1. Multi-Modal Ensembles of Regressor Chains

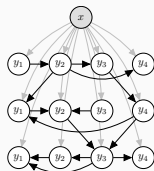
(mmERC) =

- = Ensembles of Regressor Chains +
- + Mechanism 1 (BaseEstimator level) +
- + Mechanism 2 (Ensemble level)



E. Antonenko, J. Read, *Multi-Modal Ensembles of Regressor Chains for Multi-Output Prediction*, submitted to IDA-2022 conference (Rennes, France).

2. Layered Regressor Chains (LRC)



Multi-Modal Ensembles of Regressor Chains (mmERC)

Uniform Cost Function (UCF) is an analogue of 0/1 loss for regression.

$$\text{UCF}(\delta) = \frac{1}{N} \sum_{i=1}^N \begin{cases} 0 & \text{if } \|\mathbf{y}^i - \hat{\mathbf{y}}^i\|_2 < \frac{\delta}{2}, \\ 1 & \text{otherwise.} \end{cases}$$

Goal = problem: optimize UCF.

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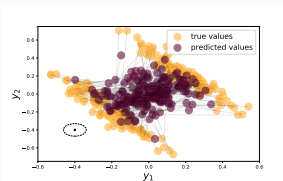
Goal = problem: optimize UCF.

	ERC	mmERC
BaseEstimator level	single round of training	train on all dataset, choose portion of data giving best predictions, retrain on this part
Ensemble level	mean for all predictions	choose the biggest cluster of predictions, take mean for this cluster only

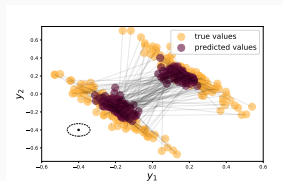
mmERC: results on a synthetic dataset

Regressor	A	B	C	D	E	Average	AvgRank
DT	0.71	0.50	0.50	0.70	0.73	0.63 ± 0.01	7.9
RF	0.84	0.47	0.45	0.78	0.84	0.67 ± 0.04	10.2
IR (dt)	0.79	0.50	0.52	0.74	0.78	0.66 ± 0.02	11.1
IR (rf)	0.86	0.47	0.47	0.79	0.87	0.69 ± 0.04	11.0
IR (svr)	0.72	0.40	0.52	0.70	0.72	0.61 ± 0.02	6.0
RC (dt)	0.74	0.50	0.51	0.70	0.72	0.63 ± 0.01	8.6
RC (rf)	0.81	0.45	0.45	0.75	0.82	0.66 ± 0.03	8.8
RC (svr)	0.70	0.40	0.51	0.67	0.71	0.60 ± 0.02	4.2
ERC (dt)	0.78	0.50	0.49	0.72	0.76	0.65 ± 0.02	8.6
ERC (rf)	0.83	0.44	0.44	0.76	0.83	0.66 ± 0.04	8.6
ERC (svr)	0.71	0.40	0.50	0.67	0.72	0.60 ± 0.02	5.0
mmERC (dt)	0.72	0.50	0.51	0.69	0.71	0.63 ± 0.01	8.2
mmERC (rf)	0.69	0.43	0.44	0.63	0.67	0.57 ± 0.02	2.2
mmERC (svr)	0.69	0.40	0.52	0.67	0.68	0.59 ± 0.02	4.6

(a) UCF results for the synthetic datasets.

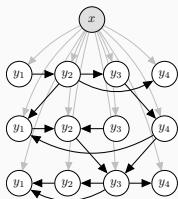


Random Forest



mmERC (RF)

Layered Regressor Chains (LRC)



Example of a single chain in ensemble:

$L = 4$ targets,

$K = 3$ layers,

$p = 2$ inter-layer connections

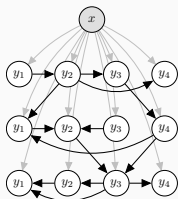
Single chain:

- Generate a random DAG in each of K layers
- Add p inter-layer connections for each two neighbour layers

Ensemble:

- Train n random layered chains
- Extract predictions from the last layer
- For each target, take mean of all predictions

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Comparing to NNs:

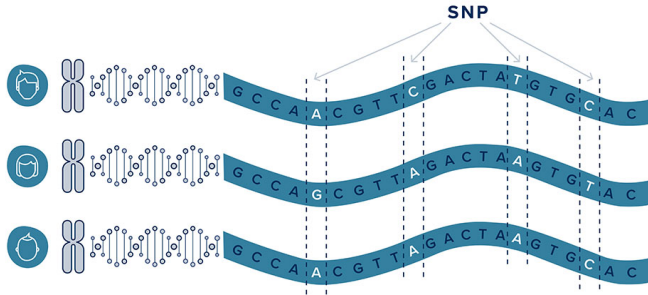
- No back-propagation \implies any BaseEstimator
- Less connections \implies lower complexity
- Work better for small datasets
- Need to train using labels from training data on each layer

mmERC + LRC: results under UCF

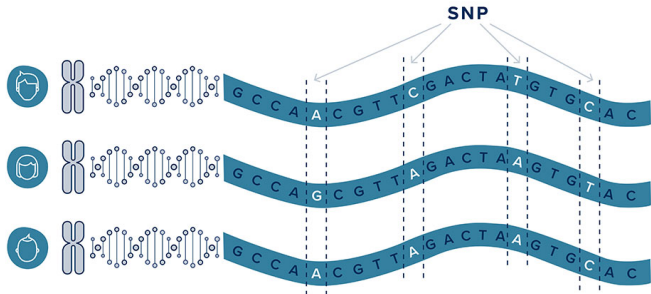
Regressor	<i>andro</i>	<i>atprd</i>	<i>atpzd</i>	<i>jura</i>	<i>oes97</i>	<i>osales</i>	<i>rf1</i>	<i>stump</i>	<i>scprf</i>	AvgRank
DT	0.72	0.33	0.34	0.48	0.88	0.94	0.01	0.66	0.18	7.39
RF	0.71	0.23	0.35	0.41	0.80	0.94	0.02	0.54	0.16	5.94
IR (dt)	0.70	0.32	0.39	0.46	0.91	0.97	0.03	0.53	0.18	8.28
IR (rf)	0.57	0.20	0.33	0.41	0.78	0.98	0.03	0.44	0.17	4.61
IR (svr)	0.64	0.70	0.86	0.60	0.93	1.00	0.10	0.46	0.23	10.67
RC (dt)	0.69	0.32	0.40	0.49	0.91	0.97	0.02	0.43	0.17	6.94
RC (rf)	0.66	0.22	0.36	0.37	0.78	0.99	0.02	0.38	0.18	5.11
RC (svr)	0.96	0.48	0.71	0.55	0.98	0.98	0.82	0.67	0.22	11.83
LRC (dt)	0.57	0.22	0.30	0.43	0.86	0.94	0.03	0.50	0.17	5.11
LRC (rf)	0.55	0.19	0.25	0.37	0.78	0.96	0.01	0.36	0.18	2.56
LRC (svr)	0.89	0.74	0.81	0.61	0.95	1.00	0.30	0.46	0.23	11.78
LRC + mmERC (dt)	0.47	0.25	0.25	0.39	0.89	0.98	0.01	0.49	0.16	4.11
LRC + mmERC (rf)	0.72	0.23	0.43	0.40	0.87	0.99	0.02	0.41	0.20	7.00
LRC + mmERC (svr)	0.94	0.98	0.96	0.67	1.00	1.00	0.57	0.71	0.25	13.67

Imputation of missing values in genomic data

Single Nucleotide Polymorphisms (SNP)



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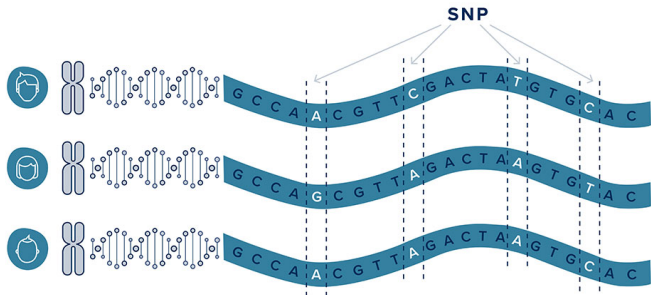
A = prevalent variant (wild-type), a = rare variant (mutant)

AA = 0

Aa = 1

aa = 2

Single Nucleotide Polymorphisms (SNP)



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AA = 0

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Features: $M = 10^5 - 10^7$
Samples: $N = 10^3 - 10^5$ } "Fat data" $X \in \{0, 1, 2\}^{N \times M}$

Genome-Wide Association Studies (GWAS)

What for:

- predicting phenotypes (i.e. diseases/traits)
- prioritizing features

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Limitations:

- lack of statistical power
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State-of-the-art:

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Limitations:

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Overcoming limitations:

Machine learning methods (e.g. linear models on graph networks / deep NNs)

Missing values

Problem:

Missing values (up to $\sim 30 - 40\%$)

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Imputation of missing values:

- can add more variants to a genetic region and increase the chances of identifying a causal variant
- facilitates the combination of results in meta-analysis when a number of studies is combined
- increases the accuracy in detecting an association signal

Imputation methods for SNP datasets

Reference-based

(fastPHASE (Scheet and Stephens, 2006), IMPUTE4 (Bycroft et al., 2017), BEAGLE (Browning et al., 2018), MACH (Li et al., 2010), etc.)

- Short chromosome segments can be inherited from a distant common ancestor
- In presence of reference panel of high quality: state-of-the-art. The accuracy is mainly determined by quality of the reference panel, and concordance of ethnicity between the data and the reference panel

Imputation methods for SNP datasets

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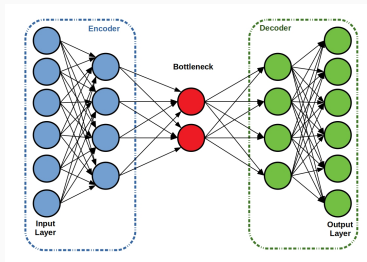
(fastPHASE (Scheet and Stephens, 2006), IMPUTE4 (Bycroft et al., 2017), BEAGLE (Browning et al., 2018), MACH (Li et al., 2010), etc.)

- Short chromosome segments can be inherited from a distant common ancestor
- In presence of reference panel of high quality: state-of-the-art. The accuracy is mainly determined by quality of the reference panel, and concordance of ethnicity between the data and the reference panel

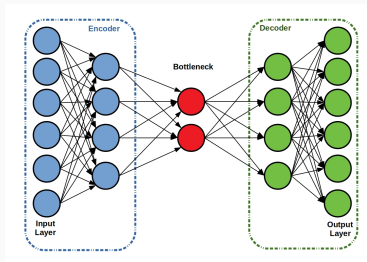
Reference-free

- Replacement with mean, median, or mode statistics
- Nearest Neighbors, Random Forests, Logistic Regression
- Autoencoders (Chen and Shi, 2019)

Autoencoders

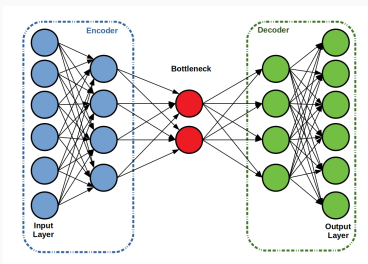


Autoencoders



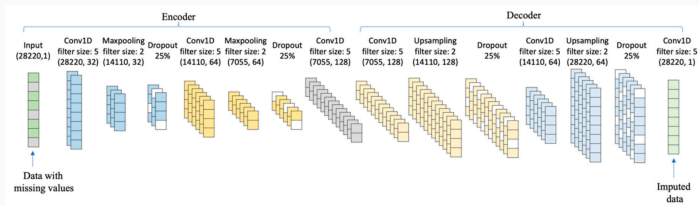
Denoising autoencoders: minimizing $L(X, g(f(\tilde{X})))$

Autoencoders



Denosing autoencoders: minimizing $L(X, g(f(\tilde{X})))$

Sparse Convolutional Denoising Autoencoders (Chen and Shi, 2019):



Chains for SNP missing values imputation

- Deep learning methods vs. fat data: ?
- Imputing missing values with a mode is known to be more effective than taking a mean
- Chaining approach can be useful in predicting missing values

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X_1	X_2
	?
?	
	?
	?

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X ₁	X ₂
	?
?	
	?
	?

X ₁	X ₂
	?
	?
	?
?	

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X ₁	X ₂
	?
?	
	?
	?

X ₁	X ₂
	?
	?
	?
?	

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X ₁	X ₂
	?
?	
	?
	?

X ₁	X ₂
	?
	?
	?
?	

X ₁	X ₂
?	

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X ₁	X ₂
	?
?	
	?
	?

X ₁	X ₂
	?
	?
	?
?	

X ₁	X ₂
?	

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X ₁	X ₂
	?
?	
	?
	?

X ₁	X ₂
	?
	?
	?
?	

X ₁	X ₂
?	

X ₁	X ₂

Preliminary results

Autoencoders idea for non-NN multi-target methods: $f(\tilde{X}) \rightarrow X$

Preliminary results

Autoencoders idea for non-NN multi-target methods: $f(\tilde{X}) \rightarrow X$

Mushroom dataset

Not SNP, but only categorical features (22 features, 8124 samples)

	Mode	DT	RF	IR(dt)	CC(dt)	MLPc
+ imputed	0.598	0.781	0.753	0.764	0.789	0.739
- changed	0.000	0.071	0.078	0.0002	0.001	0.085

Real SNP dataset (Blueberry)

Slice of data (100 features, 1000 samples)

	Mode	DT	RF	IR(dt)	CC(dt)	MLPc
+ imputed	0.769	0.734	0.796	0.773	0.787	0.821
- changed	0.000	0.229	0.184	0.002	0.004	0.135

Thank you!