# Multi-output machine learning with applications to genomics

Institut Imagine

#### Ekaterina Antonenko

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Laboratoire d'informatique, École Polytechnique, IP Paris



Introduction: Multi-output prediction

Regression: some problems

Classification: missing value imputation

#### Education

since 2020 PhD candidate: École Polytechnique Data Science and Mining (DaSciM) team, Laboratoire d'informatique (LIX), CIFRE with Digitalent, Scientific advisor: Jesse Read 2012–2014 2-year program: Moscow Bioinformatics School

2009-2014 Diploma in Mathematics: Moscow State University

#### Employment

- since 2020 École Polytechnique: Researcher, Teaching assistant (Machine learning bachelor course)
- 2017-2019 Math instructor, New York, USA
- 2013–2016 Analyst, Moscow, Russia

#### Keywords

Multi-output prediction; Missing value imputation; Explainability and interpretability; Tree methods; Probabilisctical inference.

#### Papers

- pre-print E. Antonenko, R. Beigaitė, M. Mechenich, J. Read and I. Žliobaitė, *Backward* inference in probabilistic Regressor Chains with distributional constraints.
- pre-print E. Antonenko, A.Carreño, J. Read, Autoreplicative Random Forests for missing value imputation.
- pre-print M. Konnova, E. Antonenko, J. Read, Missing value imputation for genomics data using a Sequence Based Generative Adversarial Network (SBGAN).
- ECML 2022 E. Antonenko, J. Read, *Chains of Autoreplicative Random Forests for missing value imputation in high-dimensional datasets*, [Best paper award].
  - IDA 2022 E. Antonenko, J. Read, Multi-modal ensembles of regressor chains for multi-output prediction, Advances in Intelligent Data Analysis XXI - 21st International Symposium.
- PeerJ 2016 V. Ivanenko, E. Antonenko, M. Gelfand, J. Yager, F. Ferrari, Changes in segmentation and setation along the anterior/posterior axis of the homonomous trunk limbs of a remipede (Crustacea, Arthropoda).

# Introduction: Multi-output prediction

## Definition of a multi-output problem

**Given:** Dataset  $\mathcal{D} = \{(\mathbf{x}^i, \mathbf{y}^i)\}_{i=1}^N$  of N samples:

- features  $\mathbf{x}^i = [x_1^i, ..., x_M^i]$
- outputs  $\mathbf{y}^i = [y_1^i, ..., y_L^i]$

**Goal:** Model  $f(\mathbf{X}) = \mathbf{y}$  which outputs predictions  $\hat{\mathbf{y}}^i = [\hat{y}_1^i, ..., \hat{y}_L^i]$  having  $\mathcal{D}$  observed.

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#### Example:



	x	ASE	Height	Body length	Weight
A		12,44	127,4	151	294,5
В		9,44	137,6	156	328
C		10,44	128,6	157	377
D		6,13	125,6	150	305,5
E		6,15	139	156	325
F		?	?	?	?

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**Idea:** to model these labels together in order to get better prediction performance

· Independent models

$$\hat{\mathbf{y}} = [\hat{y}_1, ..., \hat{y}_L] = [h_1(\mathbf{x}), ..., h_L(\mathbf{x})]$$



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 $h_1, ..., h_L = any single-output models$ 



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	Х	<i>y</i> 1	У2	У3	У4
F		12,32	?	?	?





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· Multi-output Decision Trees and Random Forests

$$\hat{y} = [\hat{y}_1, ..., \hat{y}_L] = [h(x)]$$

all labels are assigned simultaneously

BUT the metric to optimize is still decomposable

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 $y_3$ 

 $y_4$ 

 $y_2$ 

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## **Regression: some problems**

## Problem 1. Multi-modal distributions

Multi-output models may not work well for multi-modal distributions.

One possible reason: inadequate choice of the loss function.

Most models optimize  $MSE = \frac{1}{N} \sum_{j=1}^{N} (\mathbf{y}_j - \hat{\mathbf{y}}_j)^2$ .



Optimizing MSE does not help to exploit the dependencies between the targets.

**Uniform Cost Function (UCF)** is an analogue of 0/1 loss for regression.

$$\mathsf{UCF}(\delta) = \frac{1}{N} \sum_{i=1}^{N} \begin{cases} 0 \text{ if } \|\boldsymbol{y}^{i} - \hat{\boldsymbol{y}}^{i}\|_{2} < \frac{\delta}{2}, \\ 1 \text{ otherwise.} \end{cases}$$

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## Problem 2. Backward inference in Regressor Chains

## **Problem 2. Solution**

# Classification: missing value imputation

## Missing data

#### Why is data missing?

- Errors in sensors
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- Combining different studies
- ...

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#### Types of missingness

- Missing Completely at Random (MCAR): entirely independently of feature values
- Missing at Random (MAR): depends only on the observed feature values
- Missing Not at Random (MNAR): depends on both the observed and the unobserved feature values

#### Procedural

One procedure:

- Train a model on complete instances
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Impute randomly, then repeat:

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+ Autoreplicative Random Forests







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Which methods?

- Decision Trees, Random Forests, Extra Trees
- Classifier Chains, Multilabel k Nearest Neighbours, Random k-Labelsets, Conditional Dependency Networks, etc.

## Results: iterative ARF do converge

#### Imputation via Iterative Random Forests converges after several iterations



## Accuracy of imputation

MVR	0.01	0.05	0.1	0.2	0.3	0.01	0.05	0.1	0.2	0.3	0.01	0.05	0.1	0.2	0.3
	Mushroom [8,124 × 22]					Soybean [307 × 35]				Tumor [339 × 17]					
Complete cases	80.1%	32.3%	10.1%	0.7%	0.04%	69.7%	13.7%	1.0%	0%	0%	83.8%	38.9%	15.0%	1.2%	0.3%
MICE	0.658	0.715	0.741	0.769	0.777	0.884	0.884	0.879	0.867	0.850	0.761	0.768	0.748	0.754	0.735
itARF	0.730	0.740	0.747	0.734	0.707	0.824	0.850	0.832	0.815	0.789	0.652	0.672	0.645	0.660	0.620
pARF	0.748	0.774	0.761	0.671	0.478	0.804	0.779	0.600	-	-	0.639	0.696	0.650	0.694	0.635
itAE	0.608	0.618	0.604	0.584	0.569	0.653	0.607	0.608	0.584	0.590	0.721	0.732	0.692	0.711	0.710
pAE	0.580	0.494	0.491	0.538	0.428	0.653	0.622	0.594	-	-	0.721	0.718	0.692	0.690	0.497
itPCA	0.604	0.627	0.622	0.623	0.618	0.667	0.692	0.671	0.646	0.603	0.721	0.740	0.692	0.711	0.710
pPCA	0.600	0.587	0.578	0.537	0.441	0.655	0.639	0.620	-	-	0.721	0.671	0.688	0.626	0.411
		Vote	es [435 ×	: 16]		L	Lymphography [148 × 18]					Financial Survey [6,394 × 212]			
Complete cases	85.3%	42.2%	18.5%	1.3%	0.4%	81.8%	40.5%	14.9%	2.7%	0%	11.8%	0%	0%	0%	0%
MICE	0.768	0.795	0.771	0.768	0.782	0.750	0.679	0.665	0.648	0.651	-	-	-	-	-
itARF	0.719	0.726	0.728	0.723	0.718	0.714	0.639	0.638	0.628	0.600	0.684	0.677	0.676	0.667	0.661
pARF	0.730	0.758	0.756	0.522	0.495	0.636	0.647	0.604	0.608	_	0.633	-	-	-	-
itAE	0.697	0.563	0.602	0.578	0.570	0.700	0.474	0.485	0.448	0.487	0.626	0.617	0.616	0.604	0.596
pAE	0.638	0.546	0.600	0.524	0.488	0.679	0.514	0.563	0.611	-	0.313	-	-	-	-
itPCA	0.665	0.583	0.567	0.572	0.570	0.686	0.513	0.477	0.468	0.484	0.653	0.645	0.645	0.634	0.627
pPCA	0.595	0.499	0.567	0.507	0.453	0.693	0.536	0.562	0.502	-	0.299	-	-	-	-

- Procedural ARFs: may be powerful when enough complete instances
- MICE: as powerful as computationally expensive
- Iterative ARFs: still powerful + significantly quicker

## Time complexity

#### In theory:

- $\cdot p =$ number of features
- $\cdot n_{iter} = number of iterations$

itARF	pARF	MICE			
$\mathcal{O}(n_{iter} \cdot p)$	$\mathcal{O}(p)$	$\mathcal{O}(n_{iter} \cdot p^2)$			

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#### In practice:



## Usecase: Single Nucleotide Polymorphisms (SNP)



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- Categorical: 0 (dominant-dominant), 1 (dominant-mutant), 2 (mutant-mutant)
- High-dimensional  $(10^5 10^6)$  and low-sampled  $(10^2 10^3)$
- Ordering is important
- Missing values occur due to external mechanisms  $\implies$  MCAR

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Methods:

- reference-based (state-of-the-art for human data)
- reference-free (when reference panels are not available)

## **Chains of Autoreplicative Random Forests**



- Procedural approach: one window of size  $\Delta =$  complete instances + instances with missing values
- Chain of windows: on each step, stacking  $\nu$ windows with already imputed values as additional features
- Ensemble of chains: one forward chain, one backward chain, several random chains

## Gridsearch for parameters $\Delta$ and $\nu$



Lighter color / higher accuracy

 $\Delta: \text{ bigger fraction of missing values} \to \text{smaller size of window} \implies \\ \text{can be estimated theoretically, no need for search}$ 

 $\nu :$  may depend on problem

## Accuracy

	0.01	0.05	0.1	0.2	0.3	0.01	0.05	0.1	0.2	0.3
		Eucalypt	us [970 :	× 33,398	J					
ChARF	0.952	0.935	0.916	0.882	0.845	0.970	0.950	0.926	0.866	0.810
kNN (5/10)	0.803	0.802	0.801	0.798	0.794	0.851	0.849	0.847	0.843	0.839
mode	0.727	0.727	0.726	0.727	0.726	0.725	0.732	0.731	0.730	0.729
SVD (50/500)	0.647	0.648	0.645	0.643	0.636	0.788	0.788	0.788	0.785	0.780
MICE	-	-	-	-	-	-	-	-	-	-
	Co	lorado B	eetle [18	38 × 34,1	.86]	Arabica Coffee [596 × 4,666]				
ChARF	0.835	0.824	0.818	0.805	0.792	0.897	0.886	0.878	0.866	0.854
kNN (50/10)	0.765	0.763	0.765	0.765	0.764	0.867	0.866	0.866	0.865	0.864
mode	0.761	0.760	0.762	0.761	0.761	0.807	0.804	0.805	0.805	0.804
SVD (50/100)	0.740	0.737	0.737	0.735	0.734	0.693	0.694	0.696	0.692	0.690
MICE	-	-	-	-	-	0.757	0.741	0.724	0.689	0.664
Wheat [388 × 9,763] Coffea Canephora [119 × 45,748]										
ChARF	0.821	0.808	0.795	0.777	0.762	0.799	0.781	0.761	0.731	0.717
kNN (10/10)	0.823	0.819	0.818	0.815	0.811	0.737	0.739	0.737	0.734	0.731
mode	0.729	0.727	0.729	0.729	0.727	0.691	0.693	0.692	0.692	0.691
SVD (200/50)	0.622	0.618	0.609	0.600	0.594	0.456	0.453	0.450	0.449	0.450
MICE	0.641	0.635	0.621	0.585	0.545	-	-	-	-	-









- MICE: run with 10 neighbors for each feature, still worked only for smaller data
- Autoencoders: not taken into comparison (no complete data for training)
- Well-known methods for SNP imputation: *k* Nearest Neighbors, Single Value Decomposition

## Conclusions

- Unusual and effective usage of multi-label methods, e.g. Random Forests:
  - $\cdot$  autoreplication
  - $\cdot\,$  missing value imputation
  - $\cdot$  denoising
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- We show how probabilistic training can be easily added to the model
- ARF vs MICE: high quality and much faster
- ARF vs Autoencoders:
  - $\cdot$  no need for one-hot encoding  $\implies$  less features.
  - $\cdot\,$  lower time complexity  $\implies$  works for high-dimensional datasets
  - $\cdot\,$  no need for complete data

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- Studies for MAR and MNAR scenarios
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- How can we avoid overfitting in Iterative RF?

## Thank you!